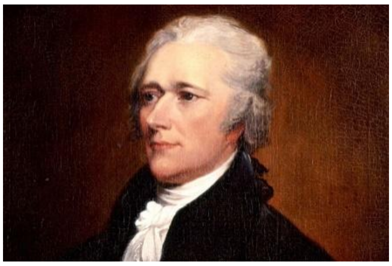
Apportioning US Representatives
2 – Different Methods

Hamilton's Method



Alexander Hamilton (1757-1804)

In the spring of 1792, a bill passed to apportion the House of Representatives, using a method proposed by one of the U.S. "Founding Fathers", **Alexander Hamilton,** and now known as Hamilton's method.

Here is the procedure:

- Compute $D = \frac{\text{U.S. Population}}{\text{Number of House seats}}$
- Then, for each state, compute State quota $= \frac{\text{State Population}}{D}$
- Assign to each state its $\mathit{State\ quota}$, rounded down.
  Keep track of fractional remainders.
- Distribute the remaining seats according to the size of the remainders, until all seats have been distributed.

1) Open the file "1_Hamilton 1792_120 seats_to be completed" and implement Hamilton's method:

   Start with the value of D in the cell L1, then complete the quota column D and so on.

<u>Spreadsheet tips:</u>

- Always use the symbol = at the beginning of a calculation or a formula
- Use the symbol $ in a formula when you want to copy and slide the formula without moving the target cell:

  For example, "$= A1/B\$1$" becomes "$= A2/B\$1$" when you copy it one cell below.
  The A has moved but not the B.
- The instruction to round down in French is "$= ARRONDI.INF(\ldots ; 0)$"
- Try to copy the values of the fractional parts (not the formulas) and sort them directly in the software (Onglet Données puis Trier dans Excel) and not by hand.

2) Write down the number of seats for each state according to Hamilton:

State	Population	Number of seats
Vermont (VT)	85,533	
New Hampshire (NH)	141,822	
Massachusetts (MA)	475,327	
Rhode Island (RI)	68,446	
Connecticut (CT)	236,841	
New York (NY)	331,589	
New Jersey (NJ)	179,570	
Pennsylvania (PA)	432,879	
Delaware (DE)	55,540	
Maryland (MD)	278,514	
Virginia (VA)	630,560	
Kentucky (KY)	68,705	
North Carolina (NC)	353,523	
South Carolina (SC)	206,236	
Georgia (GA)	70,835	
Total	**3,615,920**	

Fair enough? Not necessarily. When the bill reached the desk of President George Washington for his signature, opinions were divided among his Cabinet members (one of whom was Alexander Hamilton, the Secretary of the Treasury).
After listening to their opinions, Washington issued the **first presidential veto in U.S. history**:

Gentlemen of the House of Representatives:

I have maturely considered the act passed by the two Houses entitled "An act for an apportionment of Representatives among the several States according to the first enumeration" and I return it to your House, wherein it originated, with the following objections:

First. The Constitution has prescribed that Representatives shall be apportioned among the several States according to their respective numbers, and there is no one proportion or divisor which, applied to the respective numbers of the States, will yield the number and allotment of Representatives proposed by the bill.

Second. The Constitution has also provided that the number of Representatives shall not exceed 1 for every 30,000, which restriction is by the context and by fair and obvious construction to be applied to the separate and respective numbers of the States; and the bill has allotted to eight of the States more than 1 for every 30,000.

Jeffersons's Method

Thomas Jefferson (1743-1826)

Ten days after the veto, Congress passed a new method of apportionment, now known as Jefferson's Method in honor of its creator, **Thomas Jefferson**, also one the "Founding Fathers" and the third president of the United States.

Here is the procedure:

- Compute $D = \frac{\text{U.S. Population}}{\text{Number of House seats}}$
- Decrease D by a certain value d so that when
  State quota $= \frac{\text{State Population}}{D-d}$
  is rounded down for every state, these values add to the correct number of House seats.

1) Which of the two objections made by George Washington is obviously addressed by this method?

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2) Open the file "2_Jefferson1792_to be completed" and implement Jefferson's method: start with the value of D in cell K1, then complete quota column D (don't forget to take into account the d in cell K3, even if its value is 0 for now), the apportionment result in column F and the number of people per representative in column I.
3) By trying several possible values, find the minimum value of d so that the 120 seats are distributed according to Jefferson's method.
   $d =$ ………………
   Notice the number of people per representative in the last column. Does it address the second objection of George Washington? ……………………………………………….
   ………………………………………………………………………………………..

4) To address this second objection, the number of seats in the House was lowered to 105. Change this value in cell D2 and find the new minimum d so that the 105 seats are distributed.
   $d =$ …………………

5) Open the file "3_Hamilton 1792_105 seats_to be completed", implement Hamilton's method again with 105 seats and compare the results with Jefferson's method. Are there any differences? And if so, which states are concerned, and what are the differences?
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However, Jefferson's method is not free of any criticism or problems.

It has a **bias towards states with a large population**:

If representatives were divisible, Virginia would get around 18.31 seats and Delaware would get 1.61 seats out of the total 105 seats, and Jefferson's method (as opposed to Hamilton's) proposes to give 19 representatives to Virginia and only 1 to Delaware.

By the way, Jefferson's home state was… Virginia (surprise!)

There is a simple explanation for this bias. The method works by lowering the divisor D by some d until the rounding fits the specified number of seats. But lowering the divisor causes the quotient to grow at a faster rate if the dividend is higher. For example,

$\frac{600,000}{47,000} \approx$ …………… and $\frac{200,000}{47,000} \approx$ ……………

While $\frac{600,000}{40,000} =$ …………… and $\frac{200,000}{40,000} =$ ……………

Lowering the divisor by 7,000 in each case raises the quotient by ……… in the case of a large population but only by ………… in the case of a smaller one.

Look at the following table:

Year	**New York** Pop.	Quota	Representatives	**Delaware** Pop.	Quota	Representatives
1790	331,589	9.62	10	55,540	1.61	1
1800	577,805	16.66	17	61,812	1.78	1
1810	953,043	26.20	27	71,004	1.95	2
1820	1,368,775	32.50	34	70,943	1.68	1
1830	1,918,578	38.60	40	75,432	1.52	1
1840	2,428,919	34.05	35	77,043	1.08	1
	Totals	**157.63**	**163**	**Totals**	**9.62**	**7**

6) How many representatives did New York have in total in the $1790 - 1840$ period? …….
   How does it compare to the number it "should" have had? …………………………….
   ………………………………………………………………………………………
   What about Delaware in the same period? ……………………………………………
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It gets more unfair than that:

7) How were New York's quotas rounded compared to those of Delaware?

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8) Do you notice something bizarre in the years 1820 and 1830?

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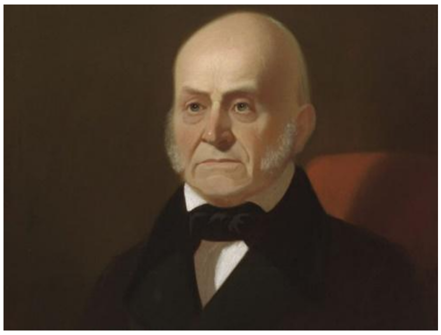
In fact, Jefferson's method repeatedly breaks what is called the **"Quota Rule"**:

> Let a state's Quota $= \frac{\text{State Population}}{\text{U.S. population}} \times$ Number of House seats
>
> Then the number of representatives for each state should be its quota, either rounded up or down.

Adam's Method



John Quincy Adams (1767-1848)

The winds of change were blowing again in 1832. **John Quincy Adams**, the 6th President of the United States from 1825 to 1829, and now a Representative from Massachusetts, proposed an **inverse of Jefferson's method**:

> - Compute $D = \frac{\text{U.S. Population}}{\text{Number of House seats}}$
> - Increase D by a certain value d so that when
> $State\ quota = \frac{\text{State Population}}{D+d}$
>   is rounded up for every state, these values add to the correct number of House seats.

1) In what sense is it the inverse of Jefferson's method? What changed?

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Let's compare these two methods with the data from the 1832 census.
The number of House seats is now 240 and more states have joined the Union.

2) Open the file "4_Adams1832_to be completed"
   - Find the minimum value of d in cell H3 so that the 240 seats are distributed according to Jefferson's method: $d =$ ………………
   - Write the formulas in columns J and L to implement Adams' method (it is just a few changes away from Jefferson's)
   - Now find the minimum value d in cell M3 so that the 240 seats are distributed according to Adams' method: $d =$ ………………

3) Compare how the seats are attributed regarding the size of the states in the two methods. What is the bias in Adams's method?

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We can use the same simple explanation for this bias: The method works by increasing divisor D by some d until the rounding fits the specified number of seats. But increasing the divisor causes the quotient to decrease at a faster rate if the dividend is higher. For example,

$\frac{600{,}000}{47{,}000} \approx$ …………… and $\frac{200{,}000}{47{,}000} \approx$ ……………

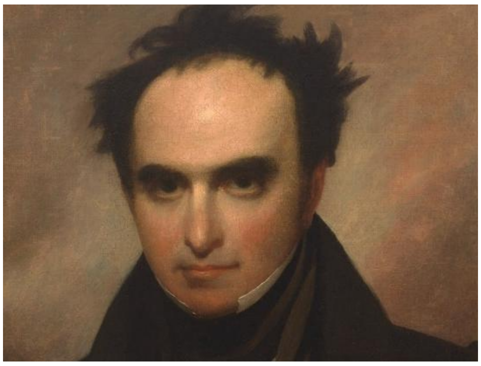
While $\frac{600{,}000}{54{,}000} \approx$ …………… and $\frac{200{,}000}{54{,}000} \approx$ ……………

Increasing the divisor by 7,000 in each case decreases the quotient by ……… in the case of a large population but only by ………… in the case of a smaller one.

By the way, **Adams's method is the one that is used in France** to apportion seats for "députés" at the National Assembly among departments.

Webster's Method

While Adams was pursuing this new procedure, Senator **Daniel Webster**, also of Massachusetts, proposed yet another alternative. It is remarkable that it was not until the 1830s that a serious proposal was put forth suggesting the use of ordinary rounding :



Daniel Webster (1782-1852)

- Let $D = \frac{\text{U.S. Population}}{\text{Number of House seats}}$
- Adjust D by a certain value d (positive or negative) so that when $State\ quota = \frac{\text{State Population}}{D+d}$ is rounded for every state, these values add to the correct number of House seats.

1) Open the file "5_Webster1832_to be completed"
Find the minimum value of d in cell H3 so that the 240 seats are distributed according to Webster's method: $d =$ ………………
2) Compare the results to those from Jefferson's and Adams's methods. Is there any bias (towards large or small states) in Webster's method? Can you explain it?

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Finally, Representative James K. Polk of Tennessee, later the 11th President of the United States, threw another idea into the mix: let's just go back to using Jefferson's method and use a divisor of 48,000. After he and others investigated many other possible divisors, it was discovered that using 47,700 instead of 48,000 would cause the quotas of the states of Kentucky, Georgia, and New York to advance past the next whole number, thus giving these three states an extra Representative each. From a political point of view, the fact that those three states collectively held about one-fourth of the House seats at the time sealed the deal: despite a flurry of proposals, **Jefferson's method was again used to apportion the House in 1832**.

The growing discomfort with Jefferson's method reached a peak in 1842, after the numbers from the 1840 Census had been finalized. After noticing the advantage gained by Kentucky, Georgia, and New York when the divisor was changed from 48,000 to 47,700, a mad search for divisors began in the Congress. More than 30 different divisors were proposed in the House within the range from about 50,000 to 62,000. Cooler heads in the Senate prevailed, however. The Senate proposed the first and only reduction in the size of the House in history. Not only that, it proposed scrapping Jefferson's method. The proposal passed, **Webster's method was used in 1842**.

Impact

Webster's time was short-lived, however. **In the 1850s it was Hamilton's method**, the one vetoed by Washington, that was adopted. In fairness to Webster, the two methods did agree on the 1852 apportionment.

The 1870s saw a new twist in the apportionment wars. In the apportionment of 1872, the House size was set to 292. Hamilton's method was legally in place. Yet the actual apportionment approved by Congress differed in four states from the Hamilton apportionment. New York was assigned 33 seats, Illinois 19, New Hampshire 3, and Florida 2. But Hamilton's method would have given New York 34, Illinois 20, New Hampshire 2, and Florida 1.

Why is this such a big deal? Because in the closely contested presidential election of 1876, Samuel Tilden won the state of New York while his opponent, Rutherford Hayes, won the other three. Hayes beat Tilden in the Electoral College by a vote of 185 to 184.

1) How many representatives would Samuel Tilden and Rutherford Hayes have had if Hamilton's method had been strictly followed? What would have happened?

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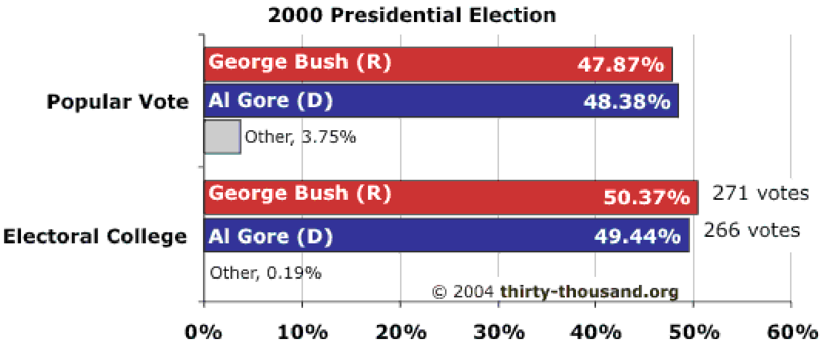
For reasons that will be explained later, **Congress voted to replace Hamilton with Webster in 1902. And in 1941, yet another method (studied in the next document) was adopted and has been used ever since.**

Now let's jump forward to the Presidential election of 2000. In the Electoral College, George W. Bush defeated Al Gore by a tally of 271 to 266. Had the Congress used Jefferson's method to apportion the House after the 1990 census, Al Gore would have garnered 271 electoral votes and become the President. Even more intriguingly, had Hamilton's method been in place, the Electoral College vote would have been tied at 269 and the election thrown to the House of Representatives for resolution. **Methods of apportionment do have practical consequences!**



2) Knowing the bias of Jefferson's method, in what kind of states was Al Gore the leading candidate?

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