The Monty Hall Problem

Rules

Congratulations! You're on a TV show and you have mastered all the questions!

You now stand in front of 3 doors.

Behind one of them is an expensive sports car; behind the other two there is a goat.

- The host (Monty Hall) asks you to pick a door where you think the sports car is.
- But he doesn't open this door yet. Instead, he opens another door and shows you a goat (he knows the location of the sports car).
  You are now faced with two doors, one with a goat behind and one with a car behind.
- He then gives you a chance to change your choice of door if you like: you can switch your selection to the other unopened door or keep your original choice.
- He finally opens the door you have chosen and you win what is behind.

Play

1) Play the game with a friend, 10 times as the player, and 10 times as the TV host. Record your decisions in the table below:

Game #	Initial choice of door	Keep or switch	Prize won (goat or car)
1			
2			
3			
4			
5			
6			
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10			

Results

2) Collect the results of the entire class and fill in the following tables:

Initial choice	% of wins

Keep or switch	% of wins

3) What seems to be the best strategy?

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4) Justify this hypothesis the best you can, with your intuition and with a maths reasoning (think about independence of events, conditional probabilities and so on).
Prepare an oral presentation of your arguments.

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This version of the problem was published by Marilyn vos Savant in a magazine in 1991. She was known for being in the Guiness Book of World Records for Highest I.Q. (228).

Since she gave her answer, Ms. vos Savant estimates she has received 10,000 letters, the great majority disagreeing with her ("You are the goat!"). Her answer has been debated in the halls of the Central Intelligence Agency and the barracks of fighter pilots in the Persian Gulf. It has been analyzed by mathematicians at the Massachusetts Institute of Technology and computer programmers at Los Alamos National Laboratory in New Mexico.

The experts responded in force to Ms. vos Savant's column. Of the critical letters she received, close to 1,000 carried signatures with Ph.D.'s, and many were on letterheads of mathematics and science departments.

"Our math department had a good, self-righteous laugh at your expense," wrote Mary Jane Still, a professor at Palm Beach Junior College. Robert Sachs, a professor of mathematics at George Mason University in Fairfax, Va., expressed the prevailing view that there was no reason to switch doors.

"You blew it!" he wrote. "Let me explain: If one door is shown to be a loser, that information changes the probability of either remaining choice -- neither of which has any reason to be more likely -- to 1/2. As a professional mathematician, I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error and, in the future, being more careful."

Of course… she was right!

Some variations

1) Let's assume that there are now 100 doors with 99 goats and 1 sports car. Once you have chosen a door, Monty Hall opens 98 doors with goats behind to leave you again with the choice between 2 doors. Do you keep your choice or switch it?
   Calculate the winning probability of each strategy.

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2) Same question, but now there are 10 doors with 6 sports car and 4 goats. Once you have chosen a door, Monty Hall shows you a door with a goat. Do you keep your choice or switch to another one?
   Calculate the winning probability of each strategy.

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An earlier version of the problem appeared in Martin Gardner's 1959 column for Scientific American:

"Three men (A, B, and C) are in separate cells and sentenced to death. The governor has selected one of them at random to be pardoned. The warden knows which one is pardoned but is not allowed to tell. Prisoner A begs the warden to let him know the identity of one of the two who are going to be executed.

- If B is to be pardoned, give me C's name. If C is to be pardoned, give me B's name. And if I'm to be pardoned, flip a coin to decide whether to name B or C.
- But if you see me flip a coin, you will know you are the one to be pardoned, replied the warden.
- Then don't tell me now, said A. Tell me tomorrow morning.

The warden, who knows nothing about probability theory, decides that if he follows the procedure suggested by A, it will give A no help whatever in estimating his survival chances. So next morning, he tells A that B is going be executed.

After the warden leaves, A smiles to himself at the warden's stupidity. There are now only two equally probable outcomes, either C is going to be pardoned or himself, so his probability of surviving has gone up from $\frac{1}{3}$ to $\frac{1}{2}$.

Prisoner A secretly tells prisoner C what happened, and C is equally overjoyed with the news because he figured, with the same reasoning used by A, that his own survival chances had also risen to $\frac{1}{2}$. Did the two men reason correctly?"

If you prefer, there is yet another version written as a poem:

"Awaiting the dawn sat three prisoners wary,
A trio of brigands named Tom, Dick and Mary.
Sunrise would signal the death knell of two,
Just one would survive, the question was who.

Young Mary sat thinking and finally spoke.
To the jailer she said, "You may think this is a joke
But it seems that my odds of surviving 'til tea,
Are clearly enough just one out of three.

But one of my cohorts must certainly go.
Without question, that's something I already know.
Telling the name of one who is lost,
Can't possibly help me. What could it cost?"

The shrivelled old jailer himself was no dummy,
He thought, "But why not?" and pointed to Tommy.
"Now it's just Dick and I" Mary chortled with glee,
"One in two are my chances, and not one in three!"

Imagine the jailer's chagrin, that old elf,
She'd tricked him, or had she? Decide for yourself.

3) What do you think? Explain your reasoning.

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